REFLECTION OF A PLANE-POLARIZED WAVE FROM A FREE SURFACE IN A STRAIN-HARDENING ELASTIC-PLASTIC MEDIUM

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The propagation of waves in an unbounded ideal elastic-plastic medium was examined in [1–4].

The behavior and laws of reflection of elastic waves were studied well and presented in [5, 6].

The problem of wave reflection in a hardening elastic-plastic medium is examined below. A plane-polarized equivoluminal shock wave propagates in a hardening elastic-plastic half-space which is assumed to be free of stresses at the boundary. The incident wave forms an angle φ with the boundary plane.

Analytical expressions are obtained for the stressed and deformed states in the plastic regions behind the front of the reflective wave. Calculations performed on a digital computer allowed us to determine the extent of these regions and changes of these regions as a function of the angle of incidence φ and parameters of material hardening.

1. Let us examine a shock wave in the form of a step which propagates in an elastic-plastic linearly hardening body. Ahead of the wave front the material is assumed to be at rest ($v_i = 0$, $\sigma_{ij} = 0$, $e_{ij}^p = 0$). Behind the wave front $v_1 = v_2 = 0$, $v_3 = V$.

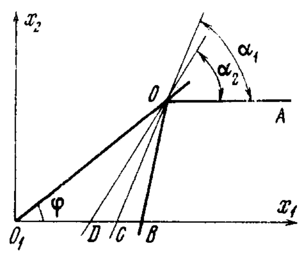

Fig. 1

Here v_i is the velocity of displacements, σ_{ij} are the stresses, e_{ij}^p are the plastic deformations and V=const.

The axis x_3 is assumed to be parallel to the line of intersection of the free surface (plane) with the front of the incident wave OA. The axis x_1 is orthogonal to x_3 and parallel to the plane of the incident wave (Fig. 1).

From conservation of momentum it follows that behind the wave front OA we have for components of stress

$$\sigma_{13} = 0, \qquad \sigma_{23} = -\sqrt{\mu\rho}\, V \tag{1.1}$$

Here ρ is the density of the material and μ the modulus of elasticity.

The material behind the front of the incident wave is in the elastic state if $|\sigma_{23}| = |\sqrt{\mu\rho}\, V| \leqslant k$, where k is the yield stress in simple shear.

For $|\sqrt{\mu\rho}\, V| > k$ (this is possible in the case of a sufficiently intensive shock on the half-space) the material behind the wave front at the initial moment of time is in the plastic state. Subsequently the front of the plastic wave releases an elastic precursor on which $|\sigma_{23}| = |\sqrt{\mu\rho}\, V| = k$ in the case of a plane wave. If the body has a free surface, the elastic precursor is first to reach it. Consequently we will assume in the following that

$$\mu\rho\, V^2 \leqslant k^2 \tag{1.2}$$

Then the plastic deformations are equal to zero behind the front of the incident wave. If the material after the reflection of the wave from the free surface remains elastic, the reflected wave will also have the form of a step. The vector of the normal to the reflected wave OB has the components

$$\nu_1 = \sin 2\varphi, \qquad \nu_2 = -\cos 2\varphi \tag{1.3}$$

Behind the front of reflected wave OB we obtain from conservation of momentum

$$\sigma_{13} = \sqrt{\mu\rho}\,[v_3]\sin 2\varphi, \quad \sigma_{23} = -\sqrt{\mu\rho}\,V - \sqrt{\mu\rho}\,[v_3]\cos 2\varphi, \quad v_3 = V - [v_3] \tag{1.4}$$

The normal to the free surface has the components $n_1 = \sin\varphi$ and $n_2 = -\cos\varphi$. The condition on the free surface OO_1 has the form

$$\sigma_{13}\sin\varphi - \sigma_{23}\cos\varphi = 0 \tag{1.5}$$

From this $[v_3] = -V$ and relationships (1.4) are now written in the form

$$\sigma_{13} = -\sqrt{\mu\rho}\,V\sin 2\varphi, \quad \sigma_{23} = \sqrt{\mu\rho}\,V(\cos 2\varphi - 1), \quad v_3 = 2V$$

Computing the intensity of stresses behind the wave front OB, we obtain

$$\sigma_{13}^2 + \sigma_{23}^2 = 4\mu\rho V^2\sin^2\varphi \tag{1.6}$$

Since V satisfies the inequality (1.2), the maximum shear stress does not exceed k, if $\varphi \leqslant {}^1/_6\pi$. If $\varphi > {}^1/_6\pi$, then the material behind the wave front OB can transform into the plastic state. In this case the solution constructed above is not applicable.

2. Just as in the case of the elastic material it is possible in the search for a solution of the elastic-plastic body is written in the form

$$\sigma_{11} = \sigma_{12} = \sigma_{22} = \sigma_{33} = 0, \quad e_{11} = e_{12} = e_{22} = e_{33} = 0, \quad v_1 = v_2 = 0$$

and that the quantities σ_{13}, σ_{23} and v_3 do not depend on x_3.

The system of equations describing the behavior of a hardening elastic-plastic body is written in the form

$$\begin{aligned} &\sigma_{i3,i} - \rho v_3^{\cdot} = 0, \qquad (k + r\varkappa)e_{i3}^{p\cdot} = (\sigma_{i3} - qe_{i3}^p)\,\varkappa^{\cdot} \\ &\varkappa^{\cdot} = (e_{i3}^{p\cdot}e_{i3}^{p\cdot})^{1/2}, \qquad \sigma_{i3}^{\cdot} = \mu v_{3,i} - 2\mu e_{i3}^{p\cdot} \end{aligned} \tag{2.1}$$

Here $i = 1, 2$. The rule of summation over recurring indices is adopted. The dot indicates partial differentiation with respect to time, r and q are parameters of hardening.

In the study of reflection of waves from a free surface we can assume that σ_{i3}, e_{i3}^p v_3 and $\varkappa$ depend only on

$$\alpha = \operatorname{arctg}\frac{x_2 - ct}{x_1 - ct\operatorname{ctg}\varphi}, \qquad c = \sqrt{\mu/\rho} \tag{2.2}$$

In this connection the system of equations (2.1) assumes the form

$$\begin{aligned} &\sigma_{13}' - \operatorname{ctg}\alpha\,\sigma_{23}' + \sqrt{\mu\rho}\,(\operatorname{ctg}\varphi - \operatorname{ctg}\alpha)\,v_3' = 0 \\ &(\sigma_{i3} - qe_{i3}^p)\,\varkappa' - (k + r\varkappa)\,e_{i3}^{p'} = 0 \end{aligned} \tag{2.3}$$

$$\begin{aligned} &(\sigma_{13}' + 2\mu e_{13}^{p'})(\operatorname{ctg}\varphi - \operatorname{ctg}\alpha) + \sqrt{\mu\rho}\,v_3' = 0 \\ &(\sigma_{23}' + 2\mu e_{23}^{p'})(\operatorname{ctg}\varphi - \operatorname{ctg}\alpha) - \sqrt{\mu\rho}\,\operatorname{ctg}\alpha\,v_3' = 0 \\ &\varkappa' = (e_{i3}^{p'}e_{i3}^{p'})^{1/2} \end{aligned} \tag{2.4}$$

Here the prime indicates the derivative with respect to α.

Substituting the values of $\varkappa'$ from (2.4) into (2.3), we obtain

$$(\sigma_{i3} - qe_{i3}^p)(\sigma_{i3} - qe_{i3}^p) = (k + r\varkappa)^2 \tag{2.5}$$

The relationship (2.5) determines the form of the surface of loading. It is assumed that there are two mechanisms of hardening: the kinematic and isotropic mechanism.

Differentiating equations (2.5) with respect to α, we find

$$(\sigma_{i3} - qe_{i3}^p)(\sigma_{i3}' - qe_{i3}^{p'}) - r(k + r\varkappa)\varkappa' = 0 \tag{2.6}$$

The system of equations (2.3) and (2.6) has the nontrivial solution σ_{i3}', $e_{i3}^{p'}$, v_3' under the condition that

$$\begin{gathered}[\operatorname{ctg}\alpha\,(\sigma_{13} - qe_{13}^p) + (\sigma_{23} - qe_{23}^p)]^2 - (k + r\varkappa)^2 \times \\ \times [(\operatorname{ctg}\varphi - \operatorname{ctg}\alpha)^2 (a + 1) - a(\operatorname{ctg}^2\alpha + 1)] = 0 \\ a = (r + q)/2\mu \geqslant 0\end{gathered} \tag{2.7}$$

Satisfying the condition (2.5) through the substitution

$$\begin{aligned}\sigma_{13} - qe_{13}^p &= -(k + r\varkappa)\cos\psi \\ \sigma_{23} - qe_{23}^p &= -(k + r\varkappa)\sin\psi\end{aligned} \tag{2.8}$$

the relationship (2.7) is transformed into

$$\cos(\alpha - \psi) = \left((1 + a)\frac{\sin^2(\alpha - \varphi)}{\sin^2\varphi} - a\right)^{1/2} = \eta(\alpha) \tag{2.9}$$

The relationship (2.9) determines ψ as a function of α. Solving Eqs. (2.3) and (2.6) for the condition (2.9), we obtain

$$k + r\varkappa = C\eta^{-\beta}(\alpha)\exp\left\{-\beta\int_{\alpha_1-\varphi}^{\alpha-\varphi}\left(\frac{\sin^2\varphi - \sin^2\alpha}{\sin^2\alpha - b\sin^2\varphi}\right)^{1/2} d\alpha\right\} \tag{2.10}$$

$$b = \frac{a}{1 + a} = \frac{r + q}{2\mu + r + q}, \qquad \beta = \frac{r}{2\mu + r + q}, \qquad \beta < b$$

The meaning of α_1 will become apparent in the course of subsequent analysis, C is a constant of integration. The remaining unknown quantities σ_{i3}, e_{i3}^p, v_3 are expressed in terms of $\varkappa$ and ψ in the form

$$\begin{aligned}e_{13}^p &= \frac{1}{2\mu + r + q}\int_{\alpha_1}^{\alpha}\frac{(k + r\varkappa)\cos\psi\,(\eta' + \sin(\alpha - \psi))}{\cos(\alpha - \psi)}\,d\alpha + C_1 \\ e_{23}^p &= \frac{1}{2\mu + r + q}\int_{\alpha_1}^{\alpha}\frac{(k + r\varkappa)\sin\psi\,(\eta' + \sin(\alpha - \psi))}{\cos(\alpha - \psi)}\,d\alpha + C_2 \\ v_3 &= \frac{2}{\sqrt{\mu\rho}\,\sin\varphi}\int_{\alpha_1}\frac{(k + r\varkappa)\sin(\alpha - \varphi)(\eta' + \sin(\alpha - \psi))}{\sin 2(\alpha - \psi)}\,d\alpha + C_3\end{aligned} \tag{2.11}$$

and through Eqs. (2.8); C_1, C_2, C_3 are constants of integration.

3. In determination of the solution for the problem of wave reflection (1.1) in an elastic-plastic material the free surface is a source of a whole packet of waves. This is a neutral shock wave which can propagate only with the velocity $c = \sqrt{\mu/\rho}$ and therefore coincides with the location of the reflected shock wave OB in the elastic case. On this wave the plastic deformations are continuous.

Behind the front of this wave (neutral region: $\sigma_{i3} = \text{const}$, $e_{i3}^p = \varkappa = 0$) the stresses are determined from Eq. (1.4) where the intensity of the wave $[v_3]$ should be determined from the condition of plasticity, so that

$$[v_3] = -V(\cos 2\varphi \pm \sqrt{z^2 - \sin^2 2\varphi}), \quad z^2 = k^2 / \mu\rho V^2 \qquad (3.1)$$

The components of stress take the form

$$\sigma_{13} = -\sqrt{\mu\rho}\, V \sin 2\varphi (\cos 2\varphi \pm \sqrt{z^2 - \sin^2 2\varphi}) \qquad (3.2)$$
$$\sigma_{23} = -\sqrt{\mu\rho}\, V[1 - \cos 2\varphi (\cos 2\varphi \pm \sqrt{z^2 - \sin^2 2\varphi})]$$

The state of stress (3.2) occurs when $2\varphi \geqslant \alpha \geqslant \alpha_1$. The angle $\alpha = \alpha_1$ determines the location of the reflected plastic stress wave OC. From the continuity of stresses for $\alpha = \alpha_1$, we obtain that α_1 must satisfy the system of equations

$$z \cos\psi = \sin 2\varphi (\cos 2\varphi \pm \sqrt{z^2 - \sin^2 2\varphi})$$
$$z \sin\psi = 1 - \cos 2\varphi (\cos 2\varphi \pm \sqrt{z^2 - \sin^2 2\varphi}) \qquad (3.3)$$
$$\cos(\alpha_1 - \psi) = \eta(\alpha_1)$$

The plastic wave OC propagates with the velocity $c_1 < c$. The velocity c_1 can be determined from Brewster's law

$$c_1 \sin\varphi = c \sin(\alpha_1 - \varphi) \qquad (3.4)$$

On the other hand, the velocity can be determined from the system of equations (2.1) at the discontinuities for the condition that $e_{i3}^p = \varkappa = 0$ and with application of kinematic conditions of coincidence of the first order. In this manner we obtain that

$$c_1 = c\sqrt{1 - (\sigma_{i3} \nu_i^{(1)})^2 k^{-2} (1 + a)^{-1}} \qquad (3.5)$$

Here σ_{i3} satisfy (3.2) and $\nu_i^{(1)}$ are components of the vector normal to the front of the wave OC ($\nu_1^{(1)} = \sin\alpha_1$, $\nu_2^{(1)} = -\cos\alpha_1$).

It is easy to show that taking into account (3.5) the relationship (3.4) is equivalent to system (3.3).

For $\alpha_1 \geqslant \alpha \geqslant \alpha_2$ solutions (2.8), (2.10) and (2.11) are applicable. Here the constants C, C_1, C_2, C_3 are determined from the condition of continuity of solution for $\alpha = \alpha_1$. We obtain

$$C = k\eta^3(\alpha_1), \quad C_1 = C_2 = 0, \quad C_3 = V[1 + (\cos 2\varphi \pm \sqrt{z^2 - \sin^2 2\varphi})] \qquad (3.6)$$

For $\alpha_2 \geqslant \alpha \geqslant \varphi$ (Fig. 1) the following trivial solution holds

$$\sigma_{13} = [qe_{13}^p - (k + r\varkappa)\cos\psi]_{\alpha=\alpha_2}, \quad \sigma_{23} = [qe_{23}^p - (k + r\varkappa)\sin\psi]_{\alpha=\alpha_2} \qquad (3.7)$$

The angle $\alpha = \alpha_2$ determines the location of the reflected plastic wave of relaxation OD.

The value of α_2 is determined from the condition of continuity of stresses on the relaxation wave OD and the boundary condition (1.5). We obtain

$$\left(\frac{a}{1+a} - \beta\right) \int_{\alpha_1}^{\alpha} \frac{(k + r\varkappa)\sin(\psi - \varphi)(\eta' + \sin(\alpha - \psi))}{\cos(\alpha - \psi)} d\alpha - (k + r\varkappa)\sin(\psi - \varphi) = 0 \qquad (3.8)$$

Knowing α_2, the velocity c_2 for the propagation of wave OD is determined from the relationship

$$c_2 \sin\varphi = c \sin(\alpha_2 - \varphi)$$

The value of α_1 for $z = 1$ (on reflection of the elastic precursor) were computed on a digital computer for various values of a and are presented in Table 1. For each value of a and φ in Table 1 two values are given. The upper value corresponds to the plus sign in Eqs. (3.3) and the lower value to the minus sign. In Eq. (2.9) the plus sign is always selected for the root, because only in this case $\alpha_2 > \varphi$.

Table 1

a	30°	40°	50°	60°	70°	80°	90°
0	56.56	79.36	98.69	60.00	84.53	109.74	135.00
	53.79	79.14	99.14	112.91	122.46	129.48	135.00
0.1	56.83	79.41	98.84	87.59	89.22	112.06	136.36
	54.65	79.23	99.21	113.32	123.24	130.59	136.36
0.2	57.05	79.46	98.96	95.27	93.34	114.15	137.61
	55.28	79.31	99.26	113.68	123.94	131.58	137.61
0.3	57.25	79.50	99.06	99.77	96.97	116.06	138.75
	55.78	79.37	99.31	114.00	124.58	132.50	138.75
0.4	57.42	79.53	99.14	102.79	100.15	117.81	139.80
	56.18	79.42	99.36	114.29	125.17	133.34	139.80
0.5	57.57	79.56	99.21	105.00	102.96	119.42	140.77
	56.51	79.46	99.39	114.55	125.70	134.12	140.77
0.6	57.70	79.58	99.26	106.68	105.43	120.90	141.67
	56.78	79.49	99.43	114.79	129.19	134.84	141.67
0.7	57.82	79.61	99.31	108.02	107.63	122.26	142.51
	57.02	79.53	99.46	115.01	126.66	135.52	142.51
0.8	57.93	79.62	99.35	109.11	109.59	123.53	143.30
	57.23	79.56	99.49	115.21	127.08	136.15	143.30
0.9	58.03	79.64	99.39	110.01	111.34	124.70	144.04
	57.40	79.58	99.51	115.39	127.48	136.74	144.04
1.0	58.12	79.66	99.43	110.77	112.91	125.81	144.74
	57.56	79.61	99.53	115.56	127.85	137.29	144.74
10	59.62	79.93	99.91	118.80	135.65	150.13	163.24
	59.61	79.93	99.91	118.96	136.49	151.72	153.24
10^2	59.96	79.99	99.99	119.88	139.54	158.66	174.35
	59.96	79.99	99.99	119.88	139.56	158.72	174.35
10^3	59.99	79.99	100.00	119.98	139.95	159.89	178.34
	59.99	79.99	100.00	119.98	139.95	159.89	178.34

Table 2

a	30°	40°	50°	60°	70°	80°	90°
0	56.56	72.77	87.55	100.83	113.38	124.54	135.00
0.1	56.83	73.22	88.21	101.89	114.40	125.88	136.36
0.2	57.05	73.66	88.88	102.80	115.50	127.09	137.61
0.3	57.25	74.05	89.48	103.60	116.48	128.19	138.75
0.4	57.42	74.38	90.02	104.33	117.38	129.20	139.80
0.5	57.57	74.69	90.50	105.00	118.21	130.13	140.77
0.6	57.70	74.96	90.94	105.60	118.97	131.00	141.67
0.7	57.82	75.20	91.33	106.16	119.67	131.81	142.51
0.8	57.93	75.43	91.70	106.68	120.33	132.55	143.30
0.9	58.03	75.62	92.04	107.16	120.94	133.27	144.04
1.0	58.12	75.80	92.34	107.61	121.50	133.92	144.74
10	59.63	79.12	98.27	116.92	134.71	150.76	163.24
10^2	59.96	79.89	99.80	119.63	139.32	158.51	174.35
10^3	59.99	79.99	99.99	119.98	139.95	159.90	178.34

Values of α_2 for $z = 1$ and various a are presented in Table 2. The values of α_2 were computed from Eq. (3.8) by the method of successive approximations. As a preliminary step the equation $\sin(\psi - \varphi) = 0$ was solved. From this the value of α_2 was computed and then the value of the integral in Eq. (3.8) was determined. To find the second approximation, the following equation was solved

$$(k + r\varkappa) \sin(\psi - \varphi) = I[a(1 + a)^{-1} - \beta]$$

where I is the value of the integral for α_2, determined from the first approximation, etc.

Computations showed that all subsequent approximations practically agree with the first approximation. Consequently, in Table 2 only one value of α_2 is presented which corresponds to the signs plus and minus in Eqs. (3.3).

In Figs. 2 and 3 values of α_1^+, α_1^- and α_2 are shown for $a = 0$ and $a = 1$. Analogous graphs are applicable in all other cases.

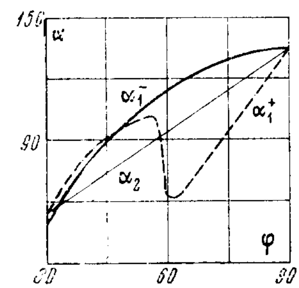

Fig. 2

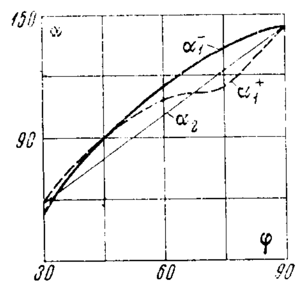

Fig. 3

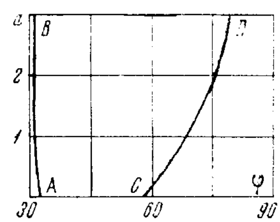

Fig. 4

We note that it follows from Tables 1 and 2 that for increasing a all curves get closer together. For $a = 100$ and $a = 1000$ the curves practically coincide and are equal to 2φ.

We note that the values α_1 determined from Eqs. (3.3), and the quantities α_2 computed from (3.8) must satisfy the inequality $\varphi \leqslant \alpha_2 \leqslant \alpha_1 \leqslant 2\varphi$, because in the opposite case the relaxation wave will overtake the stress wave.

The conditions $\varphi \leqslant \alpha_2 \leqslant 2\varphi$ and $\varphi \leqslant \alpha_1 \leqslant 2\varphi$ are always satisfied.

Let $\alpha_1 = \alpha_2$, then

$$\begin{aligned} 4(1 + a)\{\sin^2\varphi\,[1 + \sin\varphi\,(1 + (\cos 2\varphi \pm \cos 2\varphi))]^2 - \\ - \cos^4\varphi\,(1 - (\cos 2\varphi \pm \cos 2\varphi))^2\} = \\ = \sin^2 2\varphi\,(1 - (\cos 2\varphi \pm \cos 2\varphi))^2 \end{aligned} \tag{3.9}$$

Equation (3.9) determines the dependence of φ on a in the cases where $\alpha_1 = \alpha_2$. In Fig. 4 two branches are constructed. These correspond to solutions of Eq. (3.9) when the signs plus and minus are selected.

From data presented in Tables 1 and 2 it follows that if the point (a, φ) is located to the left of the curve AB, then the condition $\alpha_2 \leqslant \alpha_1$ is satisfied only when the plus sign is selected in Eqs. (3.3).

If the point (a, φ) is located to the right of the curve CD, then the condition $\alpha_1 \geqslant \alpha_2$ is satisfied only when the minus sign is selected.

We note that if in Eqs. (3.3) the minus sign is applicable, then surfaces of strong discontinuity do not exist in the packet of reflected waves.

If the point (a, φ) is located between the curves AB and CD, then both solutions can be realized.

In the solutions constructed above it was assumed that in the packet of reflected waves there can be only one shock wave OB which is neutral, i.e. the plastic deformations on the front of this wave are continuous. In the general case with plane-polarized motion in a linearly-hardening medium plastic shock waves can exist and have fronts on which the plastic deformation suffers a discontinuity. For the existence of a plastic shock wave it is necessary that the following relationship holds ahead of the wave front

$$s_{ij} = qe_{ij}^{p} + \frac{[c_i]\nu_j + [c_j]\nu_i}{\sqrt{[c_k][c_k]}}(k + r\varkappa) \qquad (3.10)$$

Here s_{ij} are components of the stress deviator.

In the case which is under examination the relationship (3.10) can be represented in the form

$$\sigma_{i3} = qe_{i3}^{p} + (k + r\varkappa)\nu_i^{(3)} \qquad (3.11)$$

Here $\nu_i^{(3)}$ are components of the vector normal to the front of the shock wave ($\nu_1^{(3)} = \sin\alpha_3$, $\nu_2^{(3)} = -\cos\alpha_3$). The angle $\alpha = \alpha_3$ determines the location of the shock wave. We will show that the conditions of existence of wave (3.11) cannot be satisfied in the case of reflection from a free surface.

We can show that the shock wave $\alpha = \alpha_3$ propagates with the velocity

$$c_3 = \sqrt{\mu_1/\rho}, \qquad \mu_1 = \mu a(1 + a)^{-1}$$

From the relationship (3.4) written for the shock wave $\alpha = \alpha_3$ we obtain the equation for the determination of its position

$$\cos(\psi - \alpha_3) = \eta(\alpha_3) = 0 \qquad (3.12)$$

Just as in the case of the solution analyzed above, the waves OB, OC, OD propagate ahead of the front of the shock wave. The position of waves OB and OC remains unchanged. The position of the relaxation wave OD must now be determined from the condition (3.11) which in combination with relationship (2.8) can be represented in the form

$$\sin(\alpha_2 - \alpha_3) = \cos(\psi - \alpha_2) \qquad (3.13)$$

Here $\alpha = \alpha_2$ determines the location of the relaxation wave OD.

It is evident from (3.13) that the relaxation wave OD either coincides with the elastic shock wave because $\alpha_2 = \alpha_3$ is a solution of this equation, or it is determined (after some transformations) from the following equation taking into account (3.12):

$$(\sin^2\varphi - a - 1)\,\mathrm{tg}(\alpha_2 - \varphi) = (\sin^2\varphi + a + 1)\,\mathrm{tg}(\alpha_3 - \varphi) \qquad (3.14)$$

However, the values α_2 determined from (3.14) do not belong to the interval $[\varphi, 2\varphi]$ and therefore they must be discarded.

Let us examine the first root $\alpha_2 = \alpha_3$. It was shown above that for $\alpha_1 \geqslant \alpha \geqslant \alpha_2$ the solution (2.8), (2.10), (2.11), (3.6) is valid. The relationship (2.10) can be represented in the form

$$k + r\varkappa = k\left(\frac{\eta(\alpha_1)}{\eta(\alpha)}\right)^{\beta} \exp\left\{-\beta\int_{\alpha_1}^{\alpha}\left(\frac{1 - \eta^2(\alpha)}{\eta^2(\alpha)}\right)^{1/2} d\alpha\right\} \qquad (3.15)$$

Let us examine the quantity $k + r\varkappa$ for $\alpha = \alpha_3$. The improper integral

$$\int_{\alpha_1}^{\alpha_3}\left(\frac{1 - \eta^2(\alpha)}{\eta^2(\alpha)}\right)^{1/2} d\alpha$$

converges. The quantity η (α_3) becomes zero according to (3.12). From this we obtain that $k + r\varkappa$ grows without bounds for $\alpha \to \alpha_3$.

From (2.8) and (2.11) it follows that σ_{i3}, e_{i3}^p, v_3 also grow without bounds for $\alpha \to \alpha_3$.

Thus, if one considers only the bounded solutions in the packet of reflected waves, then a reflected plastic shock wave does not exist.

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ON AN EFFECTIVE METHOD OF SOLVING NONCLASSICAL MIXED PROBLEMS OF THE THEORY OF ELASTICITY

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An integral equation of the first kind with a difference kernel having a logarithmic singularity is studied between finite limits. Many plane and three-dimensional mixed problems of elasticity theory and mathematical physics reduce to such integral equations.

A method is proposed for the effective solution of this equation for small values of the characteristic dimensionless parameter λ in the kernel. The principal part of the solution is extracted for small λ and the residual is sought in the form of some series of Laguerre polynomials. A certain infinite algebraic system is obtained to determine the coefficients of this series. An approximate solution of the integral equation with isolated characteristic singularities is found by truncating this system.

As illustrations, problems on the effect of a strip stamp on an elastic half-space and the impression of a stamp into an elastic strip are considered.

Certain papers of Popov [1-3] were the impetus to the development of this method.